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# Effects of Shear Deformation and Rotary Inertia on the Dynamics of a Simply Supported Anisotropic Plates Traversed by a Distributed Moving Force

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#### ABSTRACT

The effects of shear deformation and rotary inertia on the dynamics of an anisotropic plate resting on a biparametric Vlasov foundation and traversed by a distributed moving force is investigated in this work.

The Mindlin plate model is used as the basis for the mathematical model of an anisotropic plate having a varying flexural rigidity and varying density. Galerkin's weighted residual method is employed to reduce the fourth order governing partial differential equation into a set of coupled fourth order ordinary differential equation which is solved using the Laplace transform method. The method required expressing the Heaviside function that represent the distributed moving load on the structure as a Fourier sine series.

A closed form solution is obtained for the problem of anisotropic plate on a Vlasov foundation subjected to a moving distributed force. Results obtained with the aid of MATLAB programming indicate that shear modulus and rotary inertia correction factor all have significant influence on the anisotropic plate. It was observed that increasing the shear modulus and rotary inertia of the plate reduced the amplitude of displacement of the plate. Shear deformation and rotary inertia should not be neglected in models and solutions involving the dynamics of anisotropic plates traversed by moving distributed forces as this could lead to serious defects in bridges, roads, decking and machine parts.

**KEYWORDS:** Shear deformation; Rotary Inertia; Anisotropic Plate; Distributed force; Simple Support

### 1. Introduction

Plates are widely used structures with wide engineering applications in aircraft, nuclear vessels, hydraulics, bridges, and roads. There has been a great deal of research on the analysis of structures (shells, plates, and beams) with consideration for various factors such as displacements, thickness variation, stresses, curvature, the effect of surrounding media, loads and masses by authors including (Oni and Jimoh, 2016; Aiyesimi, 2000; Esen, 2015; Hull, 2016; Jia-Xing, *et al.* 2017; Jimoh, *et al.* 2017; Jimoh and Awelewa, 2017; Ozgan, 2018; Awodola, *et al.* 2019).

The influence of various parameters on structures (plates, beams, and shells) has been studied by many authors. Some of the earliest works on the influence of structural parameters on the plate was by (Mindlin, 1951) which was on the influence of rotary inertia and shear on flexural motions of isotropic elastic plates. The influence of shear deformation and rotary inertia on the natural frequencies of a thick rotating annular plate was discussed by (Cote, Atalla, & Nicolas 1997). The effect of shear deformation and rotary inertia on the natural frequencies of axially loaded beams was studied by (Koo, 2014). (Omolofe and Ogunbamike, 2014) studied the influence of prestressed foundation subgrade, rotatory inertia correction factor, the mass ratio on the flexural motions, and critical velocity. Transverse shear deformations, rotary inertia, and initial curvature effects on anisotropic plates and shells were examined by (Toorani and Lakis, 2000). The influence of shear modulus, foundation modulus, and mass ratio on elastically supported rectangular plates under concentrated moving masses and resting on the bi-parametric elastic foundation was examined by (Awodola and Omolofe, 2018) while (Nivi et al., 2019) considered the effects of shear deformation and rotary inertia on the dynamics of anisotropic plates traversed by moving concentrated load.

Furthermore, the problem of assessing the dynamic behavior of structures carrying moving loads has been almost exclusively reserved in literature to the case in which the moving loads are simplified as moving concentrated forces. It is assumed that these concentrated loads act at a point on the structure and along a single line in space as they move. That is, the moving load is modeled as a lumped load. However, in practice, it is well known that loads are actually distributed over a small segment or over the entire length of the structural member they traverse. When the moving load is distributed, the problem of investigating the load-structure interaction becomes much more complicated. Concentrated forces are mere mathematical idealization, and cannot be found in the real world, where forces are either body forces acting within the bulk of the material or within the volume.

Therefore, there is a need to study more realistic case where the moving load will be modeled as distributed

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one. The aim of the present work was to investigate the dynamic response of a square anisotropic plate with varying flexural rigidity and varying mass per unit area is considered. The anisotropic plate is simple supported on a bi-parametric Vlasov foundation and is traversed by a distributed moving mass. The influence of rotary inertia, flexural rigidity, areal density, and foundation shear deformation on the plate is examined. The design of improved plates with better qualities that are lighter and more costeffective is very important hence the use of anisotropic material and composites with high strength to weight ratio to reduce hazards due to moving masses.

## 2 Governing Equation

The equation governing the anisotropic plate traversed by a varying moving mass on a biparametric elastic Vlasov foundation is given as;

$$\begin{pmatrix} D_d(\xi,\eta)\nabla^2 - \left(\frac{\mu_d(\xi,\eta)D_d(\xi,\eta)}{hG_d} + R_0\right)\frac{\partial^2}{\partial\iota^2}\right)\nabla^2 U(\xi,\eta,\iota) + \frac{\mu_d(\xi,\eta)R_0}{hG_d}\frac{\partial^4 U(\xi,\eta,\iota)}{\partial\iota^4} + \mu(\xi,\eta)\frac{\partial^2 U(\xi,\eta,\iota)}{\partial\iota^2} + \left(k_f - G_f\nabla^2\right)U(\xi,\eta,\iota) = P_L(\xi,\eta,\iota)\left(1 - \frac{1}{g}\frac{d^2}{d\iota^2}U(\xi,\eta,\iota)\right)$$
(1)

#### 2.1 Anisotropy of the plate

Two mechanical properties of the plate are varying in different directions on the rectangular plate. The flexural rigidity of the plate  $D_d$  and the mass per unit area of the plate  $\mu_d$  are given by;

$$D_{d}(\xi,\eta) = D_{o}\left(1 - \frac{2\xi}{a} + \frac{2\xi^{2}}{a^{2}}\right)\left(1 - \frac{2\eta}{b} + \frac{2\eta^{2}}{b^{2}}\right)$$

$$\mu_{d}(\xi,\eta) = \mu_{o}\left(1 - \frac{2\xi}{a} + \frac{2\xi^{2}}{a^{2}}\right)\left(1 - \frac{2\eta}{b} + \frac{2\eta^{2}}{b^{2}}\right)$$
(2)
(3)

Where  $U(\xi, \eta, \iota)$  is the displacement of the plate  $\xi$  and  $\eta$  are spatial coordinates,  $\iota$  is the time coordinate,  $D_d$  is the variable flexural rigidity of the plate,  $\mu_d$  is the variable mass per unit area of the plate and  $D_0$  is the constant flexural rigidity of the

plate,  $\mu_0$  is the constant mass per unit area of the plate.  $R_0$  is the rotary inertia correction factor,  $G_d$  is the shear modulus, M is the mass of the load and  $v_{\xi}$  and  $v_{\eta}$  are the velocity components of the load, k and G depict the foundation stiffness and shear modulus parameter of the elastic Vlasov foundation and the load is  $P_L$  and  $\frac{d^2}{dt^2}$  is the convective acceleration.

### 2.2 Dimensionless form

The following dimensionless variables are introduced;

$$x = \frac{\xi}{a}, y = \frac{\eta}{b}, t = \frac{\iota}{t_0}$$
(4)

Where  $t_0$  will be specified and

Also, the load and convective acceleration are given as (6) and (7) respectively;

$$P_{L}(x, y, t) = MgH(x - v_{x}t)H(y - y_{0})$$

$$\frac{d^{2}}{dt^{2}} = \frac{\partial^{2}}{\partial t^{2}} + \left(2V\frac{\partial}{\partial t} + a\right)\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) + V^{2}\nabla^{2}$$
(6)

Where  $H(x - v_x t)H(y - y_0)$  is Heaviside function

Substituting (4), (5) and (6) into equation (1) and making some rearrangements yields,

$$\begin{split} \left\{ D_1 F_x F_y \left( \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \right) + D_2 F_{xy} \left( \frac{\partial^3}{\partial x^3} + \frac{\partial^3}{\partial x \partial y^2} \right) + D_2 F_{yx} \left( \frac{\partial^3}{\partial x^2 \partial y} + \frac{\partial^3}{\partial y^3} \right) + D_{12} (F_x + F_y) \\ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - b_1 F_a \left( \frac{\partial^4}{\partial x^2 \partial t^2} + \frac{\partial^4}{\partial y^2 \partial t^2} \right) \\ &- \frac{R_0}{t^2} \left( \frac{\partial^4}{\partial x^2 \partial t^2} + \frac{\partial^4}{\partial y^2 \partial t^2} \right) \\ + b_2 \frac{\mu_0}{\rho h} F_x F_y \frac{\partial^4}{\partial t^4} + b_3 \frac{\mu_0}{\rho h} F_x F_y \frac{\partial^2}{\partial t^2} \right\} U(x, y, t) \\ &+ \left\{ k_f - G_f \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right\} U(x, y, t) \\ &= P_0 H(x - v_x t) H(y - y_0) \end{split}$$

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$$-\frac{P_0}{g}H(x-v_xt)H(y-y_o)\left[b_3\frac{\partial^2}{\partial t^2} + \left(\frac{2V_0}{t_0}\frac{\partial}{\partial t} + a_0\right)\right] \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) + V_0^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) U(x,y,t)$$
(7)

where;

 $F_a$ 

$$D_{1} = \frac{D_{0}}{Eh^{3}}, D_{2} = \frac{4D_{0}}{Eh^{4}}, D_{3} = \frac{D_{2}}{h},$$

$$b_{1} = \frac{\mu_{0}D_{0}}{ED\rho h^{2}Gt_{0}^{2}}; b_{2} = \frac{R_{0}h^{6}}{DGt_{0}^{4}}, b_{3} = \frac{h^{2}}{t_{0}^{2}},$$

$$b_{3} = \frac{1}{t_{0}^{2}}, k_{f} = \frac{\kappa D}{h^{2}}; G_{f} = \frac{DG_{0}}{h^{3}}, \mu(x, y)$$

$$= \frac{\mu_{0}}{\rho h}F_{x}F_{y}, F_{xy} = (2x - 1)(1 - 2y + 2y^{2}),$$

$$F_{xy} = (2y - 1)(1 - 2x + 2x^{2})$$

$$F_{x} = (1 - 2x + 2x^{2}), F_{y} = (1 - 2y + 2y^{2}),$$

$$= (1 - 2x + 2x^{2})^{2}(1 - 2y + 2y^{2})^{2}$$
(8)

The fourth order partial differential equation in equation (7) is the dynamic plate problem of an anisotropic plate with variable flexural rigidity and variable mass per unit area acted upon by a moving distributed load which is to be investigated.

The anisotropic plate is subject to simply supported boundary conditions on all edges;

for 
$$0 \le x \le 1$$
  
 $U(0, y, t) = 0 = U(1, y, t); U_{xx}(0, y, t) = 0 = U_{xx}(1, y, t)$   
and  
for  $0 \le y \le 1$   
 $U(x, 0, t) = 0 = U(x, 1, t); U_{yy}(x, 0, t) = 0 = U_{yy}(x, 1, t)$   
(9)

With initial boundary conditions defined as follows;

$$U(x, y, t)|_{t=0} = 0; \quad U_t(x, y, t)|_{t=0} = 0;$$
  
$$U_{tt}(x, y, t)|_{t=0} = 0; \quad U_{ttt}(x, y, t)|_{t=0} = 0$$
  
(10)

#### 2.3 Method of Solution

The Galerkin's weighted residual method is used to reduce the fourth order partial differential equation

(7) into a set of coupled fourth order ordinary differential equation. The Heaviside function is expressed as a Fourier sine series and the transformed ordinary differential equation is simplified using the modified Struble's asymptotic technique for the moving mass problems and then solved using the Laplace transform method. The displacement written in the form;

$$U(x, y, t) = \sum_{m=1}^{\infty} \Lambda_m(x, y) y_m(t)$$
(11)

Where  $\Lambda_m(x, y)$  are the known eigen-functions of the plate with the same boundary conditions, obtained by considering the free vibration of rectangular plates given by;

$$\nabla^4 \Lambda_m - \frac{\mu}{D} \Omega_m^2 \Lambda_m = 0 \tag{12}$$

 $\Omega_m$ , m = 1, 2, 3, ... are the natural frequencies of the dynamic system and  $y_m(t)$  are amplitude functions which have to be solved.  $\Lambda_n(x, y)$  are assumed to be products of the function  $\phi_{ni}(x)$  and  $\phi_{nj}(y)$  which are plate functions in the direction of x and yaxes respectively. Hence,

$$\Lambda_n(x, y) = \phi_{ni}(x) \cdot \phi_{nj}(y) \tag{13}$$

Where,

$$\phi_{ni}(x) = \sin \psi_{ni} x + A_{ni} \cos \psi_{ni} x + 
B_{ni} \sinh \psi_{ni} x + C_{ni} \cosh \psi_{ni} x$$

$$\phi_{nj}(y) = \sin \psi_{nj} y + A_{nj} \cos \psi_{nj} y + 
B_{nj} \sinh \psi_{nj} y + C_{nj} \cosh \psi_{nj} y$$
(14)

Where  $A_{ni}, A_{nj}, B_{ni}, B_{nj}, C_{ni}, C_{nj}$  are constants determined by the boundary conditions,  $\psi_{ni}$  and  $\psi_{nj}$  are called modal frequencies. Since the plate under consideration has simple support at all its edges, the boundary conditions (9) is taken as

$$\phi_{ni}(0) = 0; \ \phi_{ni}(a) = 0; \ \phi_{nj}(0) = 0; \ \phi_{nj}(b) = 0;$$
(15)  
$$\frac{\partial^{2}\phi_{ni}(0)}{\partial x^{2}} = 0; \ \frac{\partial^{2}\phi_{ni}(a)}{\partial x^{2}} = 0; \ \frac{\partial^{2}\phi_{nj}(0)}{\partial y^{2}} = 0; \ \frac{\partial^{2}\phi_{nj}(b)}{\partial y^{2}} = 0$$
(16)

to obtain plate functions;

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$$\phi_{ni} = \sin n_i \pi x, \ \phi_{nj} = \sin n_j \pi y \tag{17}$$

With the constants  $A_{ni} = B_{ni} = C_{ni} = 0$  and modal frequencies;  $\psi_{ni} = n_i \pi$  and  $\psi_{nj} = n_j \pi$ 

The Heaviside function is represented using the Fourier Sine series

The Heaviside function is represented using the Fourier Sine series

$$H(x-v_{x}t) = \frac{1}{4} + \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi(x-v_{x}t)}{2n+1}$$
(18)

Applying the Galerkin's weighted residual method to (7) and rearranging yields;

$$\frac{d^4 y_m(t)}{dt^4} + \left(\alpha_1^2 - \Gamma_0 \Delta_0^* + \varepsilon_0^* \Delta_1^*\right) \frac{d^2 y_m(t)}{dt^2} + c_0 \varepsilon_0^* \Delta_{11}^{*2} \frac{d y_m(t)}{dt} + \left(\alpha_2^2 + \varepsilon_0^* \Delta_{12}^{*2}\right) y_m(t)$$
  
=  $P_n [R_{ni} + \cos \theta_x t]$ 

Where

$$\Delta_{0}^{*} = R_{a}I_{q31}I_{q32} + A_{4}I_{q41}I_{q42}; \quad \varepsilon_{0}^{*} = \frac{\varepsilon_{0}}{A_{0}I_{q01}I_{q02}}; \quad P_{n} = \frac{P_{0}I_{132}}{A_{0}n_{i}\pi I_{q01}I_{q02}};$$
$$\Delta_{1}^{*} = b_{3}I_{102}\left(\frac{I_{01}^{*}}{4} + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi v_{x}t}{2n+1}F_{1a} - \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi v_{x}t}{2n+1}F_{1b}\right)$$

(19)

$$\Delta_{11}^{*2} = I_{112} \left( \frac{Q_{10}}{4} + \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\cos(2n+1)\pi v_x t}{2n+1} F_{2a} - \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi v_x t}{2n+1} F_{2b} \right)$$
  
$$\Delta_{12}^{*2} = a_0 \Delta_{11}^2 + V_o^2 I_{122} \left( \frac{I_{q31}}{4} + \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\cos(2n+1)\pi v_x t}{2n+1} F_{3a} - \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)\pi v_x t}{2n+1} F_{3b} \right)$$
  
(20)

Equation (19) is the fundamental equation of the problem of the dynamics of anisotropic plate on Vlasov foundation having simple supports on all edges. The simple-simple anisotropic plate on Vlasov foundation traversed by a moving distributed force is considered. An approximate model of the moving force system is obtained when the inertia effect of the moving mass is neglected

$$(\varepsilon_{0}^{*}=0). \text{ Thus, (19) becomes;}$$

$$\frac{d^{4}y_{m}(t)}{dt^{4}} + (\alpha_{1}^{2} - \Gamma_{0}\Delta_{0}^{*})\frac{d^{2}y_{m}(t)}{dt^{2}} + \alpha_{2}^{2}y_{m}(t) = P_{n}[R_{ni} + \cos\theta_{x}t]$$
(21)

$$\frac{d^4 y_m(t)}{dt^4} + \alpha_{sf}^2 \frac{d^2 y_m(t)}{dt^2} + \alpha_2^2 y_m(t) = P_n \left[ R_{ni} + \cos \theta_x t \right]$$
(22)

Where,

$$\alpha_{sf}^2 = \alpha_1^2 - \Gamma_0 \Delta_0^* \tag{23}$$

Next (22) is subjected to Laplace transforms using initial boundary condition defined in (10);

$$L\left[\frac{d^{4}y_{p}(t)}{dt^{4}}\right] + \alpha_{sf}^{2}L\left[\frac{d^{2}y_{p}(t)}{dt^{2}}\right] + \alpha_{2}^{2}L\left[y_{p}(t)\right] = P_{n}L\left[R_{ni} + \cos\theta_{s}t\right]$$

$$(24)$$

$$L\left[y_{p}(s)\right]\left(s^{4} + \alpha_{sf}^{2}s^{2} + \alpha_{2}^{2}\right) = P_{n}\left[\frac{R_{ni}}{s} + \frac{s}{s^{2} + \theta_{x}^{2}}\right]$$

$$(25)$$

By using quadratic formula,

$$s^{4} + \alpha_{sf}^{2} s^{2} + \alpha_{2}^{2} = \left(s^{2} + w_{f1}^{2}\right) \left(s^{2} + w_{f2}^{2}\right)$$
(26)

Where

$$w_{f1}^{2} = \frac{1}{2} \left[ \alpha_{sf}^{2} - \sqrt{\alpha_{sf}^{4} - 4\alpha_{2}^{2}} \right]$$

$$w_{f2}^{2} = \frac{1}{2} \left[ \alpha_{sf}^{2} + \sqrt{\alpha_{sf}^{4} - 4\alpha_{2}^{2}} \right]$$
(27)

Also

$$\frac{1}{\left(s^{2}+w_{f1}^{2}\right)\left(s^{2}+w_{f2}^{2}\right)} = \frac{1}{w_{f1}^{2}-w_{f2}^{2}} \left(\frac{1}{\left(s^{2}+w_{f1}^{2}\right)} - \frac{1}{\left(s^{2}+w_{f2}^{2}\right)}\right)$$
(28)

Therefore, in view of (27) and (28), (25) becomes,

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$$\overline{y}_{p}(s) = \frac{P_{n}}{w_{f1}^{2} - w_{f2}^{2}} \left[ \left( \frac{R_{ni}}{s} + \frac{s}{s^{2} + \theta_{x}^{2}} \right) \left( \frac{1}{s^{2} + w_{f1}^{2}} - \frac{1}{s^{2} + w_{f2}^{2}} \right) \right]$$
(29)

An inversion of equation (29) using the convolution theorem as follows;

$$y_{p}(t) = \frac{P_{n}}{w_{f1}^{2} - w_{f2}^{2}} \left\{ \frac{R_{ni}}{w_{f2}} \int_{0}^{t} \sin w_{f2}(t-u) du - \frac{R_{ni}}{w_{f1}} \int_{0}^{t} \sin w_{f1}(t-u) du + \frac{1}{w_{f2}} \int_{0}^{t} \sin w_{f2}(t-u) \cos \theta_{x} u du - \frac{1}{w_{f1}} \int_{0}^{t} \sin w_{f1}(t-u) \cos \theta_{x} u du \right\}$$
(30)

This yield;

$$y_{p}(t) = \frac{P_{n}}{w_{f1}^{2} - w_{f2}^{2}} \left\{ \left[ \frac{R_{ni}}{w_{f1}^{2} w_{f2}^{2}} w_{f2}^{2} \cos w_{f1}t - w_{f1}^{2} \cos w_{f2}t + \left(w_{f1}^{2} - w_{f2}^{2}\right) \right] + \frac{1}{\left(w_{f1}^{2} - \theta_{x}^{2}\right)\left(w_{f2}^{2} - \theta_{x}^{2}\right)} \left[ \left(w_{f1}^{2} - \theta_{x}^{2}\right)\left(\cos \theta_{x}t - \cos w_{f2}t\right) - \left(w_{f2}^{2} - \theta_{x}^{2}\right)\left(\cos \theta_{x}t - \cos w_{f1}t\right) \right] \right\}$$

(31)

Substituting (31) into equation (11) gives;

$$U(x, y, t) = \sum_{m_{i}=1}^{n} \sum_{m_{j}=1}^{n} \frac{P_{n}}{w_{f1}^{2} - w_{f2}^{2}} \left\{ \left[ \frac{R_{ni}}{w_{f1}^{2} w_{f2}^{2}} w_{f2}^{2} \cos w_{f1} t - w_{f1}^{2} \cos w_{f2} t + \left(w_{f1}^{2} - w_{f2}^{2}\right) \right] + \frac{1}{\left(w_{f1}^{2} - \theta_{x}^{2}\right) \left(w_{f2}^{2} - \theta_{x}^{2}\right)} \left[ \left(w_{f1}^{2} - \theta_{x}^{2}\right) \left(\cos \theta_{x} t - \cos w_{f2} t\right) - \left(w_{f2}^{2} - \theta_{x}^{2}\right) \left(\cos \theta_{x} t - \cos w_{f1} t\right) \right] \right\}$$
  
$$\cdot \sin m_{i} \pi x \sin m_{j} \pi y$$

Equation (32) is now the transverse displacement for simply supported boundary conditions for the moving force problem of the dynamics of anisotropic plate on Vlasov foundation traversed by a moving distributed force.

### **3.** Results and Discussions

#### 3.1 Results

A square plate resting on a bi-parametric Vlasov foundation of length and breadth 1*m* and Young's modulus  $E = 1.9 \times 10^{10} Pa$  and poisson ratio (v) of 0.3 was used for the investigation. A load of mass M = 120g is assumed to travel at a velocity of 0.98m/s. Niyi et al. (2021)

### 1. Effect of Shear Deformation



Figure 1: Transverse displacement 2D profile of simply supported anisotropic plate for various values of shear deformation  $G_c$ 



Figure 2a: 3D diagram for the effect  $G_{c1} = 4 \times 10^4$ 

Figure 2b: 3D diagram for the effect  $G_{c1} = 8 \times 10^5$ 

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Figure 2c: 3D diagram for the effect  $G_{c1} = 1.2 \times 10^7$ 

Figure 2d: 3D diagram for the effect  $G_{c1} = 4 \times 10^7$ 

# 2. Effect of Rotatory Inertia



Figure 3: Transverse displacement 2D profile of simply supported anisotropic plate for various values of rotatory inertia Ro



Figure 4a: 3D diagram for the effect  $R_o = 0.02$ 

Figure 4b: 3D diagram for the effect  $R_o = 0.028$ 

0.5

0.4



Figure 4c: 3D diagram for the effect  $R_o = 0.04$ 

Figure 4d: 3D diagram for the effect  $R_o = 0.08$ 

### 3.2 Discussions

Figure 1 displays the transverse displacement response of a simply supported anisotropic plate

under the action of distributed forces for various values of shear deformation  $G_c$ . The figure shows that as  $G_c$  increases the deflection of the plate decreases. Surface plots Figures 2a to Figure 2d indicates a reduction in peaks as the values of the  $G_c$ 

factor increases from  $4 \times 10^4$  to  $4 \times 10^7$ . The effect of rotary inertia  $R_0$  on the transverse displacement of a simply supported anisotropic plate under the action of distributed force various values of rotary inertia  $R_0$  is display in figure 3. It is clearly show that the response amplitude of the

# 4. Conclusions

The effects of shear deformation and rotary inertia on the dynamics of an anisotropic plate resting on a Vlasov foundation and traversed by a moving distributed force was considered in this study. A solution to the problem for moving force is obtained using Laplace transform method in conjunction with convolution theory. Increasing the shear modulus of the anisotropic plate resulted in a reduction in the transverse displacement of the plate just as the displacement of the plate reduces for higher values of the rotary inertia correction factor. Thus, none of these structural parameters should be neglected in the problems involving the dynamics of anisotropic plates of moving distributed force. These structural parameters must be given the right of place and importance in the planning, construction and execution of engineering structures in order to avoid unexpected failure of the structures.

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plate decreases as the values of rotary inertia  $R_0$  increases. The peaks of the surface plots reduces from Figures 4a to 4d which indicates that higher values of the rotary inertia reduces the range of the displacement of the anisotropic plate.

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